# Fundamentals of Astrophysics - 2nd Ed.
# Exercise Solutions

Stan Owocki and Ian Berry

Solutions for all 144 Exercises in FoA textbook
Plus assorted Bonus exercises (currently 2)

Please report any corrections to owocki@udel.edu

May 19, 2025

# Contents

# Chapter 1

# Introduction

### Exercises

1. *Speeds*

   (a) What speed does a person at the equator move due to Earth's rotation? Give your answer in mi/hr, km/hr, and m/s.
   **The Earth has a radius of 6400 km or 4000 mi. Earth rotates every 24 hours or 86400 seconds. As such the speed a person moves at the equator will be the Earth's circumference over its rotation period,**

   $$V_{\rm eq} = \frac{2\pi R_e}{P_e} = \boxed{1700\,{\rm km/hr} = 1050\,{\rm mi/hr} = 470\,{\rm m/s}}.$$

   (b) What is the speed of the Earth in its orbit around the Sun? Give your answer in au/yr, km/s, mi/hr, and in terms of the fraction of the speed of light $v_{\rm orb}/c$?
   **The Earth is 1 AU,** $1.5 \times 10^8$ **km or** $9 \times 10^7$ **mi from the Sun. The Earth takes 1 yr, 8760 hours or** $3.15 \times 10^7$ **seconds to orbit the Sun. The speed of light is** $c = 3 \times 10^5$ **km/s. The Earth's orbital velocity around the sun is then,**

   $$V_{e,\rm orb} = \frac{2\pi d_{e,\odot}}{P_{e,\rm orb}} = \boxed{2\pi\,{\rm AU/yr} = 30\,{\rm km/s} = 6.5 \times 10^4\,{\rm mi/hr} = 10^{-4}c}$$

   (c) The Sun is about 28,000 ly from the center of the Milky Way, and takes about 200 million years to complete one "Galactic year". What is the speed of Sun in its orbit around the Milky Way, in km/s. In ly/yr? In terms of the fraction of the speed of light $v_{\rm orb}/c$?

   $$V_{\odot,\rm orb} = \frac{2\pi d_{\odot,\rm MW}}{P_{\odot,\rm orb}} = \boxed{260\,{\rm km/s} = 9 \times 10^{-4}\,{\rm ly/yr} \approx 0.9 \times 10^{-3}c}$$

2. *Sun*

   The Sun has a radius of about 700,000 km.

   (a) How many solar radii in 1 au? In 1 ly?

   $$\frac{\mathrm{AU}}{R_\odot} = \frac{1.5\times 10^8\,\mathrm{km/AU}}{7\times 10^5\,\mathrm{km/R_\odot}} \approx \boxed{200\,\mathrm{R_\odot/AU}}$$

   $$\frac{\mathrm{ly}}{R_\odot} = \frac{9.5\times 10^{12}\,\mathrm{km/ly}}{7\times 10^5\,\mathrm{km/R_\odot}} \approx \boxed{10^7\,\mathrm{R_\odot/ly}}$$

   (b) How many Earth radii $R_\mathrm{E}$ in one solar radius $R_\odot$?

   $$\frac{R_\odot}{R_e} = \frac{7\times 10^5\,\mathrm{km/R_\odot}}{6.4\times 10^3\,\mathrm{km/R_e}} \approx \boxed{100\,\mathrm{R_e/R_\odot}}$$

   (c) Solar neutrinos created in the Sun's core travel at very nearly speed of light but hardly interact with solar matter. How long does it take such core neutrinos to reach the solar surface?

   $$t_\mathrm{core} = \frac{R_\odot}{c} = \boxed{\frac{7}{3}\,\mathrm{sec}}$$

   How long to reach us on Earth?

   (d) What then is the solar radius in light-seconds? An astronomical unit in light minutes?

   **There are 7/3 light-seconds in a solar radius.**

   $$t_\mathrm{AU} = \frac{\mathrm{AU}}{c} = \boxed{500\,\mathrm{light-seconds} \approx 8\,\mathrm{light-minutes}}$$

3. *Moon*

   The Moon is about 240,000 miles from Earth.

   (a) What is the Earth-Moon distance in km? In light-seconds? In Earth radii $R_\mathrm{E}$? In solar radii $R_\odot$?

   **To convert from miles to kilometers, multiply by $\sim$ 1.6.**

   $$240,000\,\mathrm{mi}\times 1.6\,\mathrm{km/mi} = \boxed{384,000\,\mathrm{km}}$$

   $$\frac{d_\mathrm{em}}{c} = \frac{3.84\times 10^5\,\mathrm{km}}{3\times 10^5\,\mathrm{km/s}} = \boxed{1.3\,\mathrm{light-seconds}}$$

   $$\frac{d_\mathrm{em}}{R_\odot} = \frac{3.84\times 10^5\,\mathrm{km}}{7\times 10^5\,\mathrm{km/R_\odot}} \approx \boxed{0.55\,R_\odot}$$

(b) How many Earth-Moon distances in 1 au?

$$\frac{\mathrm{AU}}{d_{\mathrm{em}}} = \frac{1.5 \times 10^8\,\mathrm{km/AU}}{3.84 \times 10^5\,\mathrm{km/d_{em}}} \approx \boxed{400\,\mathrm{d_{em}/AU}}$$

4. *Ranking of sizes*

   In orders of magnitude, how much bigger is:

   **Look to Figure 1.1**

   (a) Sun vs. Earth? $\boxed{2}$
   (b) Galaxy vs. Sun? $\boxed{12}$
   (c) Galaxy vs. Earth? $\boxed{14}$
   (d) Galaxy vs. us? $\boxed{21}$
   (e) Light year vs. us? $\boxed{16}$
   (f) Light year vs. Sun? $\boxed{7}$

5. *Ranking of times*

   In orders of magnitude, how many:

   **Look to Figure 1.2 *Top***

   (a) Human lifetimes in the age of the universe? $\boxed{8}$
   (b) Heartbeats in the age of the universe? $\boxed{17.5}$
   (c) Days in the age of the universe? $\boxed{12.5}$
   (d) Human lifetimes since the dinosaurs? $\boxed{6}$
   (e) Heartbeats since the dinosaurs? $\boxed{15.5}$
   (f) Days since the dinosaurs? $\boxed{10.5}$

6. *Ranking of speeds*

   In orders of magnitude, how must faster is:

   **Look to Figure 1.2 *Bottom***

   (a) Airplane vs. slow walk? $\boxed{3}$
   (b) Earth's orbit vs. airplane? $\boxed{2}$
   (c) Sun's orbit vs. bicycle? $\boxed{5}$
   (d) Light vs. car? $\boxed{7}$
   (e) Supernova vs. moon? $\boxed{4}$
   (f) Apollo vs. bicycle? $\boxed{3.5}$

# Chapter 2

# Astronomical Distances

**Exercises**

1. A helium party balloon of diameter 20 cm floats 1 meter above your head.

   (a) What is its angular diameter, in degrees and radians?

   $$\alpha = \frac{s}{d} = \frac{0.2\,\mathrm{m}}{1\,\mathrm{m}} = \boxed{0.2\,\mathrm{rad} = 11.5^\circ}$$

   (b) What is its solid angle, in square degrees and steradians?

   $$\Omega = \frac{\pi R^2}{d^2} = \frac{\pi \times 0.01\,\mathrm{m}^2}{1\,\mathrm{m}^2} = \boxed{0.03\,\mathrm{sr} = 98.5\,\mathrm{deg}^2}$$

   (c) What fraction of the full sky does it cover?

   $$\frac{\Omega}{4\pi\,\mathrm{sr}} = \frac{0.03\,\mathrm{sr}}{4\pi\,\mathrm{sr}} = \boxed{2.4 \times 10^{-3}}$$

   (d) At what height $h$ would its angular diameter equal that of the Moon and Sun?
   **Note that** $\alpha_\odot \approx \alpha_{moon} \approx 0.5^\circ \approx 8.7 \times 10^{-3}\,\mathrm{rad}$**.**

   $$h = \frac{s}{\alpha} = \frac{0.2\,\mathrm{m}}{8.7 \times 10^{-3}\,\mathrm{rad}} = \boxed{23\,\mathrm{m}}$$

2. *Parallax of Mars*

   In 1672, an international effort was made to measure the parallax angle of Mars at opposition, when it was on the opposite side of the Earth from the Sun, and thus relatively close to Earth.

(a) Consider two observers at the same longitude but one at latitude of 45 degrees North and the other at 45 degrees South. Work out the physical separation $s$ between the observers given the radius of Earth is $R_E \approx 6400$ km.

**Viewed from the center of the Earth, the two observers at $\pm 45^o$ are separated by $90^o$, thus forming a right angle. The radius lines to each observer thus form the two lengths of an isosceles triangle with the observers separated by the base, with length**

$$s = \sqrt{2} R_e = \boxed{9.0 \times 10^3 \, \text{km}} .$$

(b) If the total angle shift measured is 22 arcsec, what is the distance to Mars? Give your answer in both km and au.

$$d = \frac{s}{\alpha} = \frac{9.0 \times 10^3 \, \text{km}}{22\text{arcsec}/206265 \, (arcsec/radian)} = \boxed{8.5 \times 10^7 \ \text{km}} = \boxed{0.57 \, \text{AU}} .$$

3. *Radar to Venus*

At the time when Venus exhibits its maximum elongation angle of about $47°$ from the Sun, a radar signal reflected by Venus is found to take a round trip time $\Delta t = 667\,\text{sec}$ to return to Earth. Assuming both Earth and Venus have circular orbits, and using the speed of light $c = 3 \times 10^5$ km/s, compute (in km) the Earth-Sun distance, 1 au.

**The distance between Earth and Venus can be found via *half* of the radar signal travel time:**

$$d_{ev} = \frac{3 \times 10^5 \, \text{km/s} \times 667 \, \text{s}}{2} = 10^8 \, \text{km}$$

**With this distance and the maximum elongation angle of $47°$, the Earth-Sun distance is then:**

$$d_{\text{AU}} = \frac{10^8 \, \text{km}}{\cos{(47°)}} \approx \boxed{1.5 \times 10^8 \, \text{km}}$$

4. *Parallax extension*

With a sufficiently large telescope in space, with angle error $\Delta\alpha \approx 1$ mas, for how many more stars can we expect to obtain a measured parallax than we can from ground-based surveys with $\Delta\alpha \approx 20$ mas? (Hint: What assumption do you need to make about the space density of stars in the region of the galaxy within 1 kpc from the Sun/Earth?)

**Ground-based observatories are limited a minimum parallax measurement of 20 mas. This parallax corresponds to a maximum distance of:**

$$d = \frac{s}{\alpha} = \frac{1\,\mathrm{AU}}{2 \times 10^{-2}\,\mathrm{arcsec}} = 50\,\mathrm{pc}.$$

**Current space-based observatories are limited to minimum parallax angles of 1 mas. This corresponds to a maximum distance of:**

$$d = \frac{1\,\mathrm{AU}}{10^{-3}\,\mathrm{arcsec}} = 1000\,\mathrm{pc}.$$

**Assuming a space density of stars** $n = 0.1\,\mathrm{stars/pc^3}$**, ground-based observatories can measure the distance of:**

$$(50\,\mathrm{pc})^3 \times 0.1\,\mathrm{stars/pc^3} = 12,500\,\mathrm{stars},$$

**whereas space-based observatories can measure the distance of:**

$$(1000\,\mathrm{pc})^3 \times 0.1\,\mathrm{stars/pc^3} = 10^8\,\mathrm{stars}.$$

**This means that space-based observatories can obtain measured parallaxes of** $10^8/12,500 = \boxed{8000}$ **times more stars than ground-based observatories.**

# Chapter 3

# Stellar Luminosity

**Exercises**

1. Suppose two stars have a luminosity ratio $L_2/L_1 = 100$.

   (a) At what distance ratio $d_2/d_1$ would the stars have the same apparent brightness, $F_2 = F_1$?
   $F_2 = F_1$, **or** $F_1/F_2 = 1$.

   $$\frac{F_1}{F_2} = \frac{L_1}{L_2}\left(\frac{d_2}{d_1}\right)^2 \rightarrow \left(\frac{d_2}{d_1}\right)^2 = \frac{L_2}{L_1} = 100 \rightarrow \frac{d_2}{d_1} = \boxed{10}$$

   (b) For this distance ratio, what is the difference in their apparent magnitude, $m_2 - m_1$?

   $$m_2 - m_1 = 2.5\log(F_1/F_2) = 2.5\log(1) = \boxed{0}$$

   (c) What is the difference in their absolute magnitude, $M_2 - M_1$?

   $$M_2 - M_1 = 2.5\log(L_1/L_2) = 2.5\log(10^{-2}) = \boxed{-5}$$

   (d) What is the difference in their distance modulus?

   $$(m_2 - M_2) - (m_1 - M_1) = 5\log(d_2/d_1) = 5\log(10^1) = \boxed{5}$$

2. A white-dwarf supernova with peak luminosity $L \approx 10^{10}\,L_\odot$ is observed to have an apparent magnitude of $m = +20$ at this peak.

   (a) What is its absolute magnitude $M$? (For simplicity of computation, you may take the absolute magnitude of the Sun to be $M_\odot \approx +5$.)

   $$M = 5 - 2.5\log(L/L_\odot) = 5 - 2.5\log(10^{10}) = \boxed{-20}$$

(b) What is its distance $d$ (in pc and ly).

$$m - M = 5\log(d/10\,\mathrm{pc}) \to 20 + 20 = 5\log(\mathrm{d}/10\,\mathrm{pc}) \to \mathrm{d} = \boxed{10^9\,\mathrm{pc}}$$
$$= \boxed{3.26 \times 10^9\,\mathrm{ly}}$$

(c) How long ago did this supernova explode (in Myr)?

**The supernova is located about 3.36 Gly away, which means the light took 3.26 Gyr to travel to Earth. Therefore this supernova occurred about $\boxed{\textbf{3260 Myr ago}}$.**

3. *Moon solid angle and albedo*

(a) Assuming the Moon reflects a fraction $a$ (dubbed the "albedo") of sunlight hitting it, derive an expression for the ratio of apparent brightness ($F_{\mathrm{moon}}/F_\odot$) between the full Moon and Sun, in terms of the Moon's radius $R_{\mathrm{moon}}$ and its distance from Earth, $d_{\mathrm{em}} \ll$ au.

**Since $d_{em} \ll$ au,**

$$L_{moon} \approx F_\odot\, a\, \pi R_{moon}^2 = 4\pi d_{em}^2 F_{moon}$$

**so**

$$\frac{F_{moon}}{F_\odot} = a\frac{\pi R_{moon}^2}{4\pi d_{em}^2} = a\frac{\alpha_{moon}^2}{4} = a\frac{\Omega_{moon}}{4\pi}\,.$$

(b) Derive the value of the albedo $a$ for which this ratio equals the fraction of sky subtended by the Moon's solid angle, i.e. for which $F_{\mathrm{moon}}/F_\odot = \Omega_{\mathrm{moon}}/4\pi$.

**By above, we see this require perfect reflection, i.e. $a = 1$.**

(c) Estimate the percentage correction to the above if one accounts for the finite size of the ratio $d_{\mathrm{em}}/\mathrm{au}$. Be sure to make clear if it makes the answers bigger or smaller by that percentage. For the revised part (b), comment on whether such an albedo $a$ is even possible.

**Accounting now for the extra distance of the full moon from the sun, we see that $L_{moon}$, and thus $F_{moon}$, is reduced by a factor $(\mathrm{au}/(\mathrm{au} + d_{em})]^2 \approx 1 - 2\mathrm{au}/d_{em} \approx 0.995$. This makes $F_{moon}/F_\odot$ slightly smaller, meaning the albedo to satisfy the condition from (b) would have to be $a \approx 1.05 > 1$, which is not possible.**

4. *Surface brightness of Sun and Moon*

(a) From the Sun's radius $R_\odot$ and distance of 1au, compute the Sun's angular diameter in both degree and radians.